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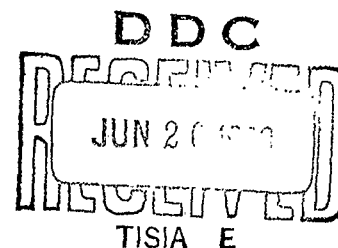
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CORRELATION METHOD FOR RECOGNIZING PATTERNS

by V. A. Kovalevskiy

- USSR -



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CORRELATION METHOD FOR RECOGNIZING PATTERNS

- USSR -

[Following is a translation of an article by V. A. Koval'skiy (Kiev), published in Zhurnal Vychislitel'noy Matematicheskoy Fiziki (Journal of Advanced Computing Mathematical Physics), Moscow, Vol 4, No 4, 1962, pp 684-694.]

The problem of recognition of patterns is one of the timely problems in cybernetics. It arouses interest especially in connection with the construction of scanning machines intended for use in electronic data processing.

The task of recognition consists of classing each pattern in one specific grade, also denominated image. The complicity of the task of recognition depends on the setup of the classes.

In this article the case is investigated, each class being set up by means of a certain standard pattern in such a way that all patterns of a given class arise from the standard pattern as an issue of distortions described through definite statistical regularities. These distortions might cover, for instance, investigations on darkening of the background, shift of the pattern, proportionate increase in the measures of the pattern, incidental darkening or lightening of certain points, denominated in the following as noises, and so on.

1. Criterion of Recognition

Each pattern can be described as a function of two variables, representing the intensity or the reflection coefficient in each point. As a specific example we shall in the following investigate patterns realized on plane paper and described by the reflection coefficient

$V(x, y)$, where x and y are the Cartesian coordinates of a certain point on the paper.

In accordance with the assumed tolerances, the pattern obtained as an issue of the k -th distortion of the standard one can be described by the function

$$p(x, y) = p^k(x, y, a_1, a_2, \dots, a_m) + r(x, y), \quad (1.1)$$

where a_1, a_2, \dots, a_m are several random parameters characterizing the distortion and $r(x, y)$ is the random function describing the sound.

One specifies in that way the superimposition of the sound out of all possible distortions, it being described as a simple additive function. That makes it easy to compute the probability serving for the interpolation of the recognition of the criterion.

In order to include a certain unknown pattern $q(x, y)$ in one of the n classes, it is necessary to compute the probability of a hypothesis that the given pattern is the result of the distortion of that or of another standard and to choose among them the greatest probability. The probabilities of the hypothesis $P_k(k)$ will present themselves as relative probabilities of the standards for the condition under which the given pattern should appear.

It is expedient to compute these probabilities by taking into account those transformations of the patterns which are generally realized with systems of revolution or scanning. In a system of this kind the beam illuminates a small section of the pattern. The illumination of the section might be unequal and is circumscribed in the general case by a certain function $g(x - x_1, y - y_1)$ where x_1 and y_1 are the coordinates of the center of the illuminated section. In the borders of the illuminated section $g(x - x_1, y - y_1) = 0$.

The light reflected from the pattern falls on a photocell, wherefore the current of the latter is proportionate to the mean coefficient of the reflection of the illuminated section D :

$$D = \iint_D p(x, y) g(x - x_1, y - y_1) dx dy.$$

With equal distribution on the range of the pattern N of points with coordinates $x_i, y_i (i = 1, 2, \dots, N)$, one might so describe the recognized patterns as the standard patterns with vectors q of N order.

In applying said transformations to equation (1.1) one obtains:

$$p_i = p^k(a_1, a_2, \dots, a_m) + r_i. \quad (1.2)$$

Let us find now the probability of the hypothesis $P_k(k)$.

According to the formula of Bayes:

$$P_k(k) = \frac{p_k(p) \cdot P(k)}{p(p)} \quad (k=1, 2, \dots, n). \quad (1.3)$$

Here $p_k(q)$ is the conditional density of the probability for pattern q appearing as a result of distortion of the k^k standard; $P(k)$ is the absolute (a priori) probability for the k standard or k image; $p(q)$ is the absolute density of the probability of the given recognition.

Let us compute first the value $p_k(q)$. So far as according to (1.2) the accidental magnitude q_1 is the total of two independent accidental magnitudes, the density of its distribution is equal to the integral beam of the distribution for $q_1(a)$ and r_1 .

Let us assume that one has to deal with a Gaussian distribution and that it has independent components. This assumption is valid if the sections illuminated by a scanning system do not overlap and have greater dimensions than the range of correlation of the noise. In such a case the following distribution of probabilities is valid for every component of the noise:

$$p(r_1) dr_1 = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{r_1^2}{2\sigma^2}\right) dr_1,$$

where σ^2 is the dispersion of the noise.

The probability that given values of the components of noise appear simultaneously is by virtue of their independence equal to the product:

$$p(r) dr_1 dr_2 \dots dr_N = \prod_{i=1}^N \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{r_i^2}{2\sigma^2}\right) dr_i = \frac{1}{(2\pi)^{N/2} \sigma^N} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^N r_i^2\right) dr_1 \dots dr_N.$$

In this way the density of the probability for the noise is:

$$p(r) = \frac{1}{(2\pi)^{N/2} \sigma^N} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^N r_i^2\right). \quad (1.4)$$

The multidimensional density of the probability of the parameters a_j is designated by $p(a)$. Then the required density of probability is:

$$p_k(q) = \frac{1}{(2\pi)^{N/2} \sigma^N} \int p(a) \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^N (q_i - p_i^k(a))^2\right\} da_1 da_2 \dots da_m. \quad (1.5)$$

Here the m -fold integral moves in the m -dimensional range of the variations of the parameters a_j .

The probabilities of the hypotheses might be computed in accordance with the equation (1.3). Thus, there is no need to compute the magnitude $p(q)$ figuring in the denominator, as one might find it on the condition of the normalization:

$$\sum_{k=1}^N P_k(k) = 1. \quad (1.6)$$

Thus:

$$P_k(k) = P(k) \int p(a) \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^N (q_i - p_i^k(a))^2\right\} da_1 \dots da_m. \quad (1.7)$$

Here the coefficient α does not depend on k and appears under conditions of normalization (1.6).

The recognition of the patterns by the method of probabilities of hypotheses can be reproduced in the following way. Let us introduce the concept of the N th space in which the values of the reflection coefficients $\{r_k\}$ of the individual sections of the pattern serve as coordinates. In such space, a certain point corresponds to each pattern. If one subjects the standard pattern to all distortions, with the exception of superposition of the noise, the corresponding point is shifted in the space describing a range in a certain m -dimensional subspace. Let us denominate it as the range of the standard.

The ranges of the various standards do not intersect one another if the distortions are such that one cannot obtain an identical design out of two different standards.

In superimposing the noise the point might pass the limits of the range of the standard. The probability that the point, as a consequence of the superimposition of the noise, fell out of the range of the k standard into the neighborhood of the point with coordinates q_k is characterized by the above-produced magnitude $p_k(q)$. As is shown in (1.4), the density of the probability of the noise is a function of $\sum_{i=1}^N r_i^2$. It is expedient in connection with that to introduce into the surface in question the distance between the two points $q^{(1)}$ and $q^{(2)}$ is determined by the magnitude

$$d = \sqrt{\sum_{i=1}^N (q_i^{(1)} - q_i^{(2)})^2}.$$

Now one can state that as a consequence of (1.5) and (1.6) the density of the probability $p_k(q)$ and the probability $P(k)$ decrease proportionately as the point recedes from the range of the k standard. One can infer by analogy with the obvious three-dimensional instance that this geometrical locus is an $(N - 1)$ dimensional hypersurface subdividing the entire surface in two ranges: on one of them prevails the probability of the k hypothesis, on the other the \bar{k} respectively.

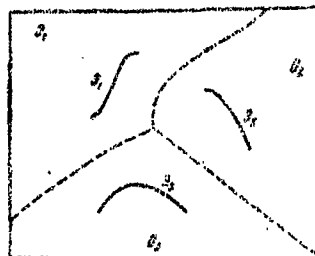


Figure 1

If one draws up such interfaces for each pair of standards, the whole surface will appear to be subdivided into n ranges, and we shall call

then subsequently ranges of images or classes of patterns. In each image range is enclosed the corresponding standard (Figure 1).

In order to recognize a pattern it suffices to determine in which of the n ranges the point of this pattern falls. The answer to this question is given by computing the probability of the hypotheses $P_k(k)$ and by comparing them.

The criterion of recognition based on the probability of the hypotheses appears to be perfect. It has, however, the deficiency that the formula (1.7) for computing the values $P_k(k)$ is complicated, wherefore it is hardly suitable for use in some kind of recording. It is interesting in connection with that to find a more simple criterion of recognition that ensures a high-level probability of correct recognition. One might find such a simplified criterion on the basis of the following considerations.

The distances between the interface of two definite standards and the ranges of those standards are virtually not equal. The relation of these distances depends as much on the peculiarities of the standard patterns manifested in the various characters of the function $\varphi_1^k(a_1, a_x, \dots, a_m)$, as on the a priori probabilities of the standards $P(k)$. However, in the case of a small noise dispersion (in comparison with the square distance to the interface) the probability is very small that a pattern near to the interface might appear, and a deliberate shift of the interface within certain limits hardly affects the probabilities of correct recognition.

Therefore, one might take in the capacity of an interface of two standards a geometrical locus of points at equal distance from the ranges of these standards. Then, to recognize a pattern, it suffices to compute its shortest distance from all the ranges of the standards:

$$\min_{\mathbf{a}} d_k = \min_{\mathbf{a}} \sum_{i=1}^n |p_i - \mathbf{a}|^2 \quad (i=1, 2, \dots, n) \quad (1.8)$$

and select the smallest of these magnitudes. Here the minimum affects all the values of the parameters of the distortions a_j . Such criterion of recognition is considerably more simple and convenient for computations.

One might estimate in the following way the probability of an incorrect recognition by this criterion: a pattern of the k class will be incorrectly recognized if the corresponding point falls beyond the limits of the range of the k image bordered by equidistant surfaces. In order to compute the probability of such an event, it is necessary to integrate the expression (1.5) by a density of probability $p_k(\rho)$ for the section of the space \bar{K} , situated outside the range of the k image. Consequently, the probability of an erroneous recognition of a pattern belonging to the image k is equal to:

$$P_k(\text{err.}) = \int_{\mathcal{R}} p_k(\rho) d\rho_1 d\rho_2 \dots d\rho_N. \quad (1.9)$$

The probability of an incorrect recognition of a pattern belonging to a particular image is equal to:

$$P(\text{err.}) = \sum_{k=1}^N P_k(\text{err.}) \cdot P(k),$$

where $P(k)$ is the a priori probability of the k image.

It is evident that the probability of an error in a particular image does not exceed the maximum error of the probabilities

$$P(\text{err.}) \leq \max_k P_k(\text{err.}).$$

Let us write the expression $P_k(\text{err.})$ after having substituted (1.5) and (1.9)

$$P_k(\text{err.}) = \frac{1}{(2\pi)^{N/2} \sigma^N} \int_{\mathcal{R}} p(\mathbf{a}) \exp \left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^N |\rho_i - \rho_i^k(\mathbf{a})|^2 \right\} d\mathbf{a}_1 \dots d\mathbf{a}_m d\rho_1 \dots d\rho_N.$$

Exchanging the sequence of integration and observing that:

$$\int_{\mathcal{R}} p(\mathbf{a}) d\mathbf{a}_1 \dots d\mathbf{a}_m = 1,$$

one might estimate:

$$P_k(\text{err.}) \leq \frac{1}{(2\pi)^{N/2} \sigma^N} \max_{\mathcal{R}} \int_{\mathcal{R}} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^N |\rho_i - \rho_i^k(\mathbf{a})|^2 \right\} d\rho_1 \dots d\rho_N. \quad (1.10)$$

Let us substitute a range of integration more convenient for computation -- to wit, with a section S of the space located outside the sphere S with a radius of $\frac{1}{2} d_{\min}$, where d_{\min} is the shortest distance between two ranges of standards in utmost proximity (see Figure 2.) The center of the sphere is in the point $\rho_i^k(\mathbf{a}^k)$, where \mathbf{a}^k is the value of the parameters of the distortion for which the integral (1.10) is at maximum. As far as such a sphere lies as a whole inside the range of the k image, the space outside the sphere comprises the space outside the range of the image. Consequently, through substitution of the range of integration, the integral of the positive density of the probability can only increase. This is valid for any k , among them that where the magnitude $P_k(\text{err.})$ is at a maximum because the chosen radius of the sphere is smaller than half of the shortest distance between the particular standards:

$$P(\text{err.}) \leq \max_k P_k(\text{err.}) < \frac{1}{(2\pi)^{N/2} \sigma^N} \int_{\mathcal{R}} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^N |\rho_i - \rho_i^k(\mathbf{a}^k)|^2 \right\} d\rho_1 \dots d\rho_N.$$

With the substitution of the variables $\mathbf{r}_1 = \rho_i - \rho_i^k(\mathbf{a}^k)$, one transfers the center of the sphere to the origin of the coordinates.

(1.11)

The expression of the right side of (1.11) represents by itself the probability that the noise vector r will be found beyond the borders of the sphere S_0 with a radius $\frac{1}{2}d_{min}$, that is, the inequality

$$P(\text{err.}) < \frac{1}{(2\pi)^{N/2} \sigma^N} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^N r_i^2 \right\}$$

will be fulfilled

$$\sum_{i=1}^N r_i^2 > \left(\frac{1}{2} d_{min} \right)^2$$

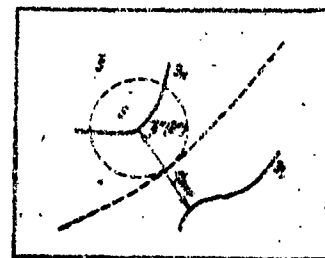


Figure 2.

One might find the probability of this inequality by means of the so-called dispersion χ^2 (see [1]). The total of the N square independent incidental magnitudes subordinate to a normal distribution with zero center and dispersion σ^2 follows for great N a rule of distribution close to the normal one with center $N\sigma^2$ and dispersion $2N\sigma^2$; therefore:

$$P(\text{err.}) < P \left\{ \sum_{i=1}^N r_i^2 > \frac{d_{min}^2}{4} \right\} \approx \Phi \left[-\frac{\frac{1}{4} d_{min}^2 - N\sigma^2}{\sqrt{2N}\sigma^2} \right]. \quad (1.12)$$

Here $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$ is the normal rule of distribution.

The obtained relation makes it simple to establish, for instance, a conclusion like: the probability of the error does not exceed 0.001 if:

$$\frac{1}{\sqrt{2N}\sigma^2} \left(\frac{1}{4} d_{min}^2 - N\sigma^2 \right) > 3.1,$$

that is,

$$d^2 < d_{min}^2 \frac{1}{4(N+3.14\sqrt{2N})}. \quad (1.13)$$

It is necessary to observe that the submitted appraisal of the probability of error is greatly overestimated and the probability of error is actually considerably lower. This is connected with the possibility of highly overstating the number of standards n in case of a number N elements of distribution, that there exist many directions in the space to which a point of the pattern might

[Subscript after P = error]

be displaced and not come close to any interface. By substituting for the actual interfaces the sphere with minimum radius, we reckon that every point at a distance from the range of that standard exceeding $\frac{1}{2}d_{\min}$ implies an intersection of the interface and induces an error.

As a result we come to the conclusion that to recognize patterns one can make successful use of the criterion of the minimum distances -- i. e., the minimum magnitude for every standard and for every possible distortion.

$$d_i^2 = \sum_{k=1}^N |p_k - p_k^*(s)|^2$$

and to investigate for which standard this minimum is smallest.

2. Recognition of Machine-Recorded Signs

By recognizing letters and numerals printed on a certain recording machine one can choose for standards shapes of signs printed under certain ideal conditions. In that case definite tolerances might be admitted concerning possible distortion in the standards. Let us suppose that a certain sign printed on the given machine is different from the standard only in its location on the paper, the average coefficient of reflection inherent to the paper and the paint, and also in the presence of different defects of printing to be considered as noise. The computed alternations of the standards can be expressed through following functions:

$$p(x, y) = ap^*(x - \xi, y - \eta) + b + r(x, y).$$

Here $p(x, y)$ is the coefficient of the reflection of the distorted pattern; $p^*(x, y)$ is the accidental function of the noise, ξ and η are accidental magnitudes characterizing the location of the standard pattern along the axes X and Y respectively; a and b are accidental values determining even darkening and even change of the contrast ratio of the picture.

By means of the parameters a and b we might describe any change in the average coefficients of the background and design reflection.

In order to get convinced that being so it is sufficient to keep in mind that a system of equations

$$\begin{cases} p_p = ap_p^* + b, \\ p_s = ap_s^* + b. \end{cases}$$

where p_p^* and p_s^* are the mean coefficients of the background and

[design for the standard and ρ_1 and ρ_2 are the same for the picture, --] is always solved concerning a and b in case $\rho_1 \neq \rho_2$.

One can write the analogous correlation for N-dimensional vectors obtained as an issue of a transformation of the pattern by a scanning device:

$$p_i = \rho_1^*(\xi, \eta) + b + r_i.$$

In accordance with the above substituted criterion, one has for recognition an unknown pattern ρ to compute n values:

$$Q_k = \min_{a, b} \sum_{i=1}^N [p_i - a \rho_1^*(\xi, \eta) - b]^2 \quad (k=1, 2, \dots, n). \quad (2.1)$$

The minima for a and b are found in a general form. Differentiating the total of (2.1) by a and b , we find:

$$\sum_{i=1}^N [p_i - a \rho_1^*(\xi, \eta) - b] = 0; \quad (2.2)$$

$$\sum_{i=1}^N [p_i - a \rho_1^*(\xi, \eta) - b] \rho_1^*(\xi, \eta) = 0. \quad (2.3)$$

The equations (2.2) and (2.3) determine the values a and b , corresponding to the most probable ρ_1 and ρ_2 for the given pattern. Therefore, the roots (2.2) and (2.3) lie for all real patterns in the range of determination of the parameters a and b and correspond to the minimum

$$Q_k^2$$

Having solved the equation (2.2) with regard to b , one obtains

$$b = \frac{1}{N} \sum_{i=1}^N p_i - a \frac{1}{N} \sum_{i=1}^N \rho_1^*(\xi, \eta).$$

Introducing the designations:

$$\frac{1}{N} \sum_{i=1}^N p_i = \bar{p},$$

Then:

$$\frac{1}{N} \sum_{i=1}^N \rho_1^*(\xi, \eta) = \bar{\rho}^*(\xi, \eta).$$

$$b = \bar{p} - a \bar{\rho}^*(\xi, \eta).$$

Substituting that in (2.3) we find a :

$$a = \frac{\sum_{i=1}^N (p_i - \bar{p}) \rho_1^*(\xi, \eta)}{\sum_{i=1}^N [\rho_1^*(\xi, \eta) - \bar{\rho}^*(\xi, \eta)] \rho_1^*(\xi, \eta)}.$$

Let introduce the designations:

$$\rho_k = \bar{\rho} - \rho_{k0},$$

$$\rho_k^2(t, \eta) = \rho^2(t, \eta) = \rho_{k0}^2(t, \eta)$$

substitute the found values a and b in (2.1). Then:

$$\alpha_{k, \min} = \min_{t, \eta} \sum_{i=1}^N \left[\rho_{k0} - 2 \frac{\sum_{i=1}^N \rho_{k0}^2(t, \eta)}{\sum_{i=1}^N \rho_{k0}^2(t, \eta) + \rho_{k0}^2(t, \eta)} \right] =$$

$$= \min_{t, \eta} \left[\sum_{i=1}^N \rho_{k0}^2 - 2 \frac{\sum_{i=1}^N \rho_{k0}^2(t, \eta)}{\sum_{i=1}^N \rho_{k0}^2(t, \eta) + \rho_{k0}^2(t, \eta)} \sum_{i=1}^N \rho_{k0}^2(t, \eta) + \right.$$

$$\left. + \left(\frac{\sum_{i=1}^N \rho_{k0}^2(t, \eta)}{\sum_{i=1}^N \rho_{k0}^2(t, \eta) + \rho_{k0}^2(t, \eta)} \right)^2 \sum_{i=1}^N \rho_{k0}^2(t, \eta) \right].$$

Let us observe that:

$$\sum_{i=1}^N \rho_{k0}^2(t, \eta) = \sum_{i=1}^N \rho_{k0}^2(t, \eta) - \rho^2(t, \eta) \sum_{i=1}^N \rho_{k0}^2 - \sum_{i=1}^N \rho_{k0}^2(t, \eta).$$

wherefore:

$$\sum_{i=1}^N \rho_{k0}^2 = 0.$$

By analogy:

$$\sum_{i=1}^N (\rho_{k0}^2(t, \eta))^2 = \sum_{i=1}^N \rho_{k0}^2(t, \eta) \rho_{k0}^2(t, \eta).$$

Let us introduce the designations:

$$\sum_{i=1}^N \rho_{k0}^2(t, \eta) = H_k,$$

$$\sum_{i=1}^N (\rho_{k0}^2(t, \eta))^2 = M_k.$$

Then:

$$\alpha_{k, \min} = \min_{t, \eta} \left(\sum_{i=1}^N \rho_{k0}^2 - 2 \frac{H_k^2}{H_k} + \frac{H_k^2}{M_k} M_k \right) = \min_{t, \eta} \left(\sum_{i=1}^N \rho_{k0}^2 - \frac{H_k^2}{M_k} \right).$$

As far as the value $\sum_{i=1}^N \rho_{k0}^2$ does not depend on k_1 , ξ and η

$$\alpha_{k, \min} = \sum_{i=1}^N \rho_{k0}^2 - \max_{t, \eta} \frac{H_k^2}{M_k}$$

and the task to find the smallest $\alpha_{k, \min}$ is equivalent to the determination of the maximum of the value $R_k \sqrt{M_k}$, or -- what is the same -- the maximum of $R_k / \sqrt{M_k}$ for all values of ξ and η and also for k .

The computation of the value $R_k / \sqrt{M_k}$ comes to the determination of scalar products of the form:

$$Q_k = \sum_{i=1}^N p_i \frac{p_{ik}^k(k, \eta)}{\sqrt{M_k}},$$

as far as

$$\sum_{i=1}^N p_{ik} p_{ik}^k(k, \eta) = \sum_{i=1}^N p_i p_{ik}^k(k, \eta) = \sum_{i=1}^N p_i^2(k, \eta)$$

and

$$\sum_{i=1}^N p_{ik}^k(k, \eta) = 0.$$

It is natural to presuppose that on the standard patterns the background has a constant coefficient of reflection in all its points and the shifts of the standard come forth provided that the design does not transgress the borders of the determination range of the pattern. In that case, the magnitudes M_k and p_i^k do not depend on the parameters of the shifts ξ and η . The value $\sum_{i=1}^N p_{ik}^k / \sqrt{M_k}$ for each standard can be computed beforehand for any disposition of the standard pattern. Then the magnitudes Q_k might be computed as scalar products of the vector of an unknown pattern by n constant vectors $p_{ik}^k / \sqrt{M_k}$. In order to find the maximum Q_k for all possible shifts it is necessary to place the components of both vectors in the shape of matrices and displace these matrices in relation to one another, computing each time the scalar product and comparing it with the other products obtained by other shifts. In order to find with these shifts for each component of a certain matrix the corresponding component in another matrix, the shifts must be made cyclically, -- i. e., a line or a column transgressing the limits of the matrix on one side, has to be transferred to the opposite side of the matrix. Instead of that, one might have matrices of different sizes.

The described algorithm of recognition has been experimentally investigated with the aid of a universal computer. The tests carried out with machine-recorded numerals checked a high reliability of recognition. With a low quality of printing, the probability of error is on the order of 10^{-4} .

3. Conclusions

In order to recognize some printed signs under conditions where the different patterns of a certain sign might differ by progressive

displacements, even changes in the coefficients of reflection of the background and design or by the presence of accidental defects, issues near to an optimum can be obtained in the following way.

One chooses as the standard a pattern as clearly printed as possible. By means of a scanning device the pattern is put on N different intersecting parts, and the elements and the mean coefficient of reflection q_i ($i = 1, 2, \dots, N$) is checked for each element. The coefficients of reflection are consequently represented for each of the n standard patterns, q_i^k , by the following relation

$$q_i^k = \frac{q_i - \frac{1}{N} \sum_{i=1}^N q_i}{\sqrt{\sum_{i=1}^N \left(q_i - \frac{1}{N} \sum_{i=1}^N q_i \right)^2}}$$

The thus obtained vectors of the normalized standards are recorded.

In order to recognize unknown patterns, its coefficients of reflection are measured by means of the same scanning device. The scalar products of the established vector are subsequently computed for all normalized vectors of the standards. For each standard one computes the scalar product many times, while the components of the vectors to be multiplied are placed in such a way as to imitate all possible shifts of the standard pattern in relation to the pattern to be recognized. Thus the number of scalar products to be computed is equal to $n \cdot w$, where n is the number of standards and w the number of shifts. One has to find the greatest scalar product among all the computed scalar products. The number of the denomination of the standard for which one has obtained the greatest scalar product is the issue of recognition.

The proposed algorithm ensures high reliability of recognition and might constitute a basis for constructing a computer.

The theoretical methods of statistical solutions for recognition have also been used heretofore (see [2], [3]), but the actual paths of distortions were not taken into consideration and the algorithms were not completed up to working order.

The described algorithm of recognition differs from other algorithms based on a computation of scalar products (see [4], [5]) to the extent that the proposed method of determining the standards is an optimum under the investigated conditions.

The idea of pattern-recognition by comparison with standards is not new [6]. It is, on the other hand easy to make sure that a direct

computation of scalar products of vectors describing patterns does not ensure correct recognition. It is necessary to subtract the average and normalize the standard patterns in this or another way.

Substantiation of this state constitutes the basic applied issue of this article.

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